

# A Complexity Reduced ML Detector on OFDM-CDM Systems in Mobile Channel

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## Abstract

Orthogonal frequency-division multiplexing (OFDM) is one of the most promising techniques for the 4-th generation mobile system. When the OFDM system is applied to cellular environment, it is required to make the transmission scheme robust to other-cell interference (OCI). The use of code division multiplexing (CDM) has been considered to mitigate the OCI in OFDM systems. In cell boundary, it is desirable to design OFDM-CDM transceiver scheme to reduce the effect of OCI. In this paper, we consider transceiver schemes for cell boundary users in the OFDM-CDM downlink. We propose a simplified maximum likelihood (ML) detection scheme that can nearly achieve full ML detection performance. In addition, a new spreading code is proposed for BPSK modulation, yielding a receiver gain of about 1dB regardless of detection schemes. Finally, the performance of the proposed schemes is verified by computer simulation.

## 1. Introduction

Broadband wireless packet access system has attracted much attention to achieve high-speed transmission capacity to support rapid increase of wireless traffic. Orthogonal frequency-division multiplexing (OFDM) is a very promising scheme for this purpose due to the simplicity of channel equalization in severe frequency selective wireless channel [1].

When the OFDM system is applied to cellular environment, it is desirable to make it robust to other-cell interference (OCI) since it is required to make the frequency reuse factor close to one to increase the system capacity. When the mobile stations are near the cell boundary region, they severely suffer from the interference from other cells. In this case, it is desirable to use a low-order modulation scheme with interference averaging capability. The use of fast frequency hopping (FH) [2] and spreading [3] schemes has been widely considered to obtain the interference averaging effect. Combined with forward error correction (FEC) coding, the FH scheme can obtain the interference averaging effect. However, the averaging effect cannot be obtained fully when the code rate is high (with weak FEC). Since the spreading scheme uses several subcarriers to transmit one information

symbol, the diversity and interference averaging effect can easily be obtained by a simple spreading operation. Thus, the OFDM-CDM can be a good choice for its simplicity and interference averaging capability [4].

In the OFDM-CDM system, the data symbol is spread over several subcarriers using orthogonal spreading codes. It is desirable to use multi-code transmission to prevent the waste of bandwidth efficiency due to the spreading. However, self-interference occurs due to the loss of orthogonality between the spreading codes in frequency selective fading channel. To alleviate this problem, several detection schemes have been proposed [5]. The use of combining scheme is widely applied with one tap equalization for its simple operation, including the maximal ratio combining (MRC), equal-gain combining (EGC), orthogonality restoring combining (ORC) and minimum mean square combining (MMSEC). The MMSEC can provide good performance but it requires the information on the power of interference.

Since the interference is very time-variant in the cell boundary, it may not be easy to estimate the power of the OCI in the next receiving packet. The maximum likelihood (ML) detection scheme provides the optimum performance but its complexity increases rapidly as the number of used orthogonal spreading codes,  $K$ , and/or the number of information bits per modulation symbol,  $c$ , increase. In this paper, we propose a complexity reduced ML detector. Assuming the use of QAM with squared signal constellation, the information bits corresponding to the real part of the symbol can be processed separately from those of the imaginary part. The computational complexity can be reduced to  $2^{K(c/2)+1}$  from  $2^{Kc}$  without loss of detection performance. We also propose a simplified ML search for complexity reduction. In addition, new spreading codes are proposed for BPSK that can significantly reduce the self-interference.

Following Introduction, the system model is described in Section II. The proposed ML schemes are presented in Section III. The performance of the proposed schemes is verified by computer simulation in Section IV. Finally, conclusions are summarized in Section V.

## 2. System model

Fig. 1 depicts the structure of considered OFDM-CDM transmission that uses  $N_c$  subcarriers for  $J$  users. Each

user symbols are spread over  $L (= N_c / J)$  subcarriers and  $K$  ( $K \leq L$ ) spreading codes can be used to multiplex each user symbol. In fact, this system is a frequency division multiple access (FDMA) system and CDM is used to merely multiplex the user symbols.

Since it is sufficient to model one user's signal in the FDMA system, the user index will be omitted for simplicity of description in what follows. The  $k$ -th user symbol  $d^{(k)}$  is spread using the spreading code  $\mathbf{c}^{(k)} = [c_1^{(k)}, c_2^{(k)}, \dots, c_L^{(k)}]^T$ .  $K$  number of data modulated spreading codes are all chip-synchronously summed, making an  $L$ -dimensional vector signal

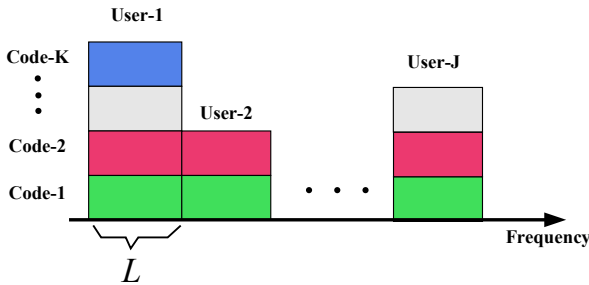
$$\mathbf{s} = [s_1, s_2, \dots, s_L]^T = \sum_{k=1}^K d^{(k)} \mathbf{c}^{(k)} \quad (1)$$

$$= \mathbf{C} \cdot \mathbf{d}$$

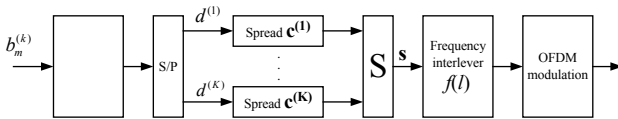
where  $\mathbf{C} = [\mathbf{c}^{(1)}, \mathbf{c}^{(2)}, \dots, \mathbf{c}^{(K)}]$  is the  $(L \times K)$  code matrix and  $\mathbf{d} = [d^{(1)}, d^{(2)}, \dots, d^{(K)}]^T$  is the data symbol vector. Here the superscript  $T$  denotes the matrix transpose operation. Then, the signal  $s_l$  is mapped to the  $f(l)$ -th subcarrier, where  $f(l)$  denotes the frequency interleaving function. Thus, the user signal is transmitted in a set of subcarriers,  $F = \{f(1), f(2), \dots, f(L)\}$ . The received signal vector  $\mathbf{r} = [r_1, r_2, \dots, r_L]^T$  after the discrete Fourier transform (DFT) can be written as

$$\mathbf{r} = \mathbf{H} \cdot \mathbf{s} + \mathbf{n} \quad (2)$$

where  $\mathbf{n} = [n_1, n_2, \dots, n_L]^T$  represents additive white Gaussian noise,  $\mathbf{H}$  is an  $(L \times L)$  matrix representing the complex channel fading, given by



(a) The structure of the OFDM-CDM signal



(b) The transmitter of the OFDM-CDM system

Fig. 1. System model

$$\mathbf{H} = \text{diag}(h_1, h_2, \dots, h_L). \quad (3)$$

Here,  $\text{diag}(\cdot)$  represents a diagonal matrix and  $h_l$  denotes the complex valued fading coefficient on the subcarrier assigned to the transmitted signal  $s_l$ . Assuming that the channel has Rayleigh fading and the frequency interleaving function is ideal,  $h_l$  can be model as an independent identically distributed (*iid*) zero-mean complex Gaussian random variable with unit variance.

Fig. 2 depicts a conventional OFDM-CDM receiver with a linear combining scheme. The received signal after the DFT is equalized using a one-tap equalizer,

$$\mathbf{u} = \mathbf{W} \cdot \mathbf{r} \quad (4)$$

where  $\mathbf{W}$  is an  $(L \times L)$  diagonal matrix whose element is the combining coefficient  $w_l$ . The best performance can be achieved using the MMSEC with [5]

$$w_l = \frac{h_l^*}{|h_l|^2 + 1/\gamma_s K} \quad (5)$$

where  $\gamma_s$  is the signal-to-noise power ratio (SNR).

The equalized signal is despread using the spreading code, yielding a soft decision value

$$v^{(k)} = \mathbf{u}^T \cdot \mathbf{c}^{(k)*} \quad (6)$$

Although the MMSEC scheme minimizes the symbol error in the presence of noise and self-interference, its performance can severely be degraded due to the self-interference as  $K$  increases. The joint detection scheme such as ML detection can be applied to alleviate the performance degradation problem.

### 3. ML detection

#### A. Conventional scheme

Let  $\mathbf{b}^{(k)} = [b_1^{(k)}, b_2^{(k)}, \dots, b_c^{(k)}]$  be the bit vector mapped onto the modulation symbol  $d^{(k)}$  and  $\mathbf{b} = [\mathbf{b}^{(1)}, \mathbf{b}^{(2)}, \dots, \mathbf{b}^{(K)}]$  be a  $Kc$ -dimensional bit sequence. The modulation symbol  $d^{(k)}$  has  $2^c$  number of signal constellation points. The ML detector searches the symbol vector maximizing the likelihood function for all possible  $2^c$ -ary symbol sequences,

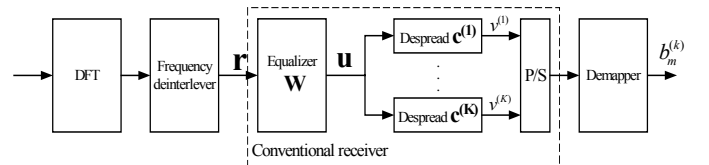


Fig. 2. The structure of conventional OFDM-CDM receiver

$$\hat{\mathbf{d}} = \arg \min_{\mathbf{d}_i} \|\mathbf{r} - \mathbf{H} \cdot \mathbf{C} \cdot \mathbf{d}_i\|^2, \quad i = 1, \dots, 2^{Kc} \quad (7)$$

where  $\mathbf{d}_i$  denotes the  $i$ -th symbol sequences. The ML detection scheme provides the optimum performance but the complexity increases rapidly as

$$\Gamma = 2^{Kc} \quad (8)$$

We consider a simplified ML detection scheme to reduce the complexity. Since the complexity depends on both  $c$  and  $K$ , we propose two detection schemes corresponding to each parameter. These schemes can be jointly combined.

### B. Proposed complexity reducing schemes

Assuming the use of QAM with squared signal constellation, (7) can be rewritten as

$$\begin{aligned} \hat{\mathbf{d}} &= \arg \min_{\mathbf{d}_i} \sum_{l=1}^L \left| r_l - h_l \sum_{k=1}^K c_l^{(k)} d_i^{(k)} \right|^2, \quad i = 1, \dots, 2^{Kc} \\ &= \arg \min_{\mathbf{d}_i} \sum_{l=1}^L \left| r_l - h_l \sum_{k=1}^K c_l^{(k)} [x_i^{(k)} + jy_i^{(k)}] \right|^2 \\ &= \arg \min_{\mathbf{d}_i} \sum_{l=1}^L \left| \frac{h_l^*}{|h_l|} r_l - |h_l| \sum_{k=1}^K c_l^{(k)} [x_i^{(k)} + jy_i^{(k)}] \right|^2 \end{aligned} \quad (9)$$

where real-valued  $x_i^{(k)}$  and  $y_i^{(k)}$  represent the real and imaginary part of symbol  $d_i^{(k)}$ , respectively. Assuming that the spreading codes are real-valued binary codes such as Walsh-Hadamard (WH) code, (9) can be rewritten as

$$\begin{aligned} \hat{\mathbf{d}} &= \arg \min_{\mathbf{d}} \left( \sum_{l=1}^L \left| \frac{h_l^* r_l}{|h_l|} - |h_l| \sum_{k=1}^K c_l^{(k)} x_i^{(k)} \right|^2 + \sum_{l=1}^L \left| \frac{h_l^* r_l}{|h_l|} - |h_l| \sum_{k=1}^K c_l^{(k)} y_i^{(k)} \right|^2 \right) \\ &= \hat{\mathbf{x}} + j\hat{\mathbf{y}} \end{aligned} \quad (10)$$

where

$$\begin{aligned} \hat{\mathbf{x}} &= \arg \min_{\mathbf{a}_i} \sum_{l=1}^L \left| \frac{h_l^* r_l}{|h_l|} - |h_l| \sum_{k=1}^K c_l^{(k)} x_i^{(k)} \right|^2 \\ \hat{\mathbf{y}} &= \arg \min_{\mathbf{b}_i} \sum_{l=1}^L \left| \frac{h_l^* r_l}{|h_l|} - |h_l| \sum_{k=1}^K c_l^{(k)} y_i^{(k)} \right|^2 \end{aligned} \quad (11)$$

Note that the information bits corresponding to the real part of the symbol can be processed separately from the imaginary part. Since  $x_i^{(k)}$  and  $y_i^{(k)}$  are  $2^{c/2}$ -ary valued, the complexity is reduced to  $2^{K(c/2)+1}$  without loss of the detection performance. For example, if  $L = K = 8$  and QPSK modulation is used (*i.e.*,  $c = 2$ ),  $\Gamma = 2^{16}$  can be reduced to  $\Gamma = 2^9$  when binary

spreading codes are used. We will call this Scheme-A.

In mobile communication channel, it may not practical to employ multi-level modulation schemes higher than 64-QAM (*i.e.*,  $c = 6$ ). Since it is practical to use a low-order modulation scheme in the cell-boundary, it is likely that  $K$  is larger than  $c$ . We consider the reduction of the ML complexity with respect to  $K$ . Fig. 3 depicts the proposed simplified ML detection called Scheme-B. The soft decision value  $v^{(k)}$  is obtained using a conventional receiver. Since the magnitude of the soft decision value can be thought as a reliability factor of the demodulated symbol, we use only  $N$  out of  $K$  symbols having small soft-decision values for ML search. The most unreliable symbols are tested by the ML detection instead of searching all the possible symbol space. Thus, the complexity of the ML detector can be reduced to  $\Gamma = 2^{cN}$  where  $N$  is an integer less than  $K$ . If this scheme is used jointly with the proposed scheme-A, the complexity can further be reduced to  $\Gamma = 2^{cN/2+1}$ . However, if  $N$  is too small, the detection performance can be degraded. This trade-off between the complexity and performance will be discussed in the next section by computer simulation.

### C. Proposed scheme for BPSK

When the proposed ML detector is directly applied to BPSK modulation, it may not be possible to provide the same performance with reduced complexity. To avoid possible problems with the use of the proposed ML detector for BPSK modulation, we use the following code as the spreading code,

$$\mathbf{c}_p^{(k)} = \begin{cases} \mathbf{c}^{(k)} & k \leq K/2 \\ e^{j\pi/2} \cdot \mathbf{c}^{(k)} & \text{otherwise} \end{cases} \quad (12)$$

where  $\mathbf{c}^{(k)}$  denotes the  $k$ -th real-valued binary spreading code such as Walsh-Hadamard and extended  $m$ -sequence. BPSK-modulated symbol  $\{d^{(1)}, d^{(2)}, \dots, d^{(K)}\}$  can be divided into two sub-sets whose elements are separately spread into real and imaginary part. Thus, we can still maintain the ML complexity of  $\Gamma = 2^{Kc/2+1}$ . In addition, the detection performance can be improved because the number of interfering codes becomes one half. This proposed

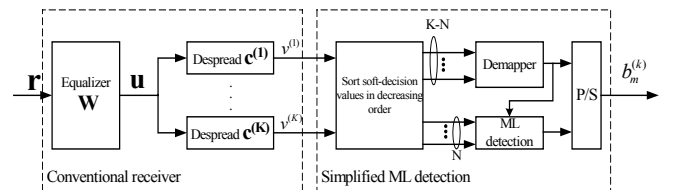


Fig. 3. The structure of the proposed simplified ML scheme

scheme can also be used jointly with the proposed scheme-B.

## 4. Simulation results

To verify the proposed detection scheme, the receiver performance is evaluated in terms of the BER in Rayleigh fading channel by computer simulation. Fig. 4 depicts the SNR loss of OFDM-CDM with QPSK modulation due to the use of multi-code when the MMSEC is used. It can be seen that the SNR loss becomes larger than 5dB as  $K$  approaches to  $L$ , which is mainly due to the self-interference.

When the ML detection scheme is applied, it may not be practical to use  $K$  larger than 8 due to the complexity. In this case, the maximum diversity order obtained from the spreading becomes 8. The diversity order higher than 8

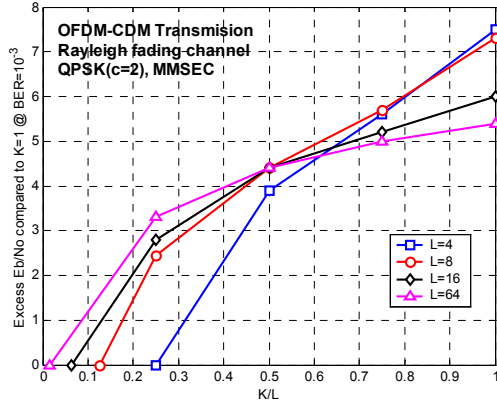


Fig. 4. The performance of MMSEC

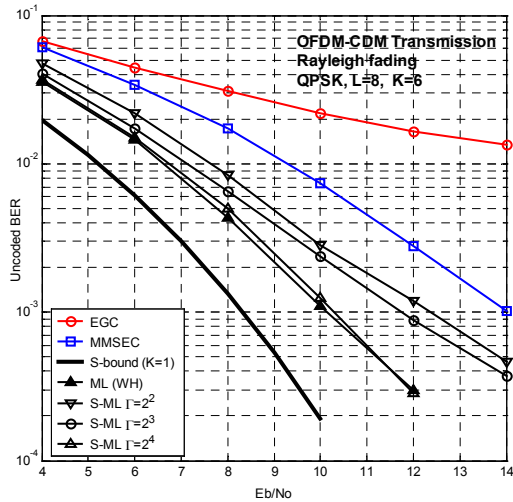


Fig. 5. BER performance of the proposed QPSK scheme

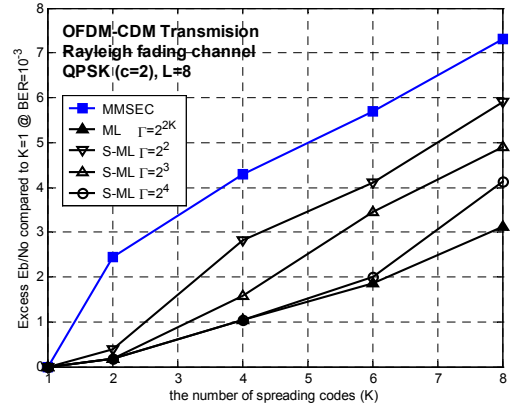


Fig. 6. The performance loss of detection schemes

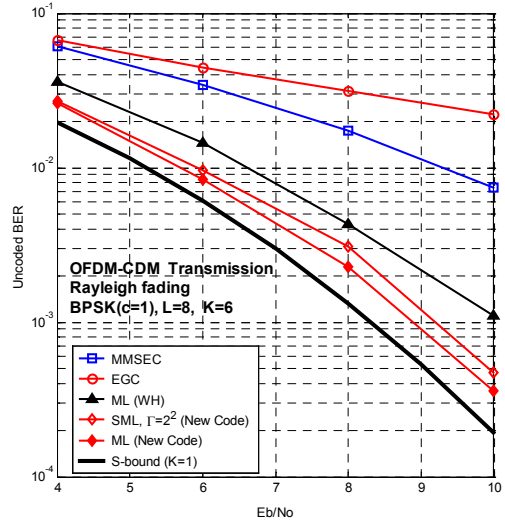


Fig. 7. BER performance of the proposed scheme (BPSK)

does not provide significant performance improvement since the diversity gain is already sufficiently obtained.

Fig. 5 depicts the BER performance of various detection schemes as function of  $E_b / N_0$  when QPSK modulation is employed without coding and  $K=6$ . ‘S-ML’ means the joint use of the proposed scheme A and B. It can be seen that S-ML with  $\Gamma = 2^4$  provides near ML detection performance, yielding about 3.7dB performance enhancement compared to the MMSEC at  $\text{BER}=10^{-3}$ . The performance improvement of the S-ML with  $\Gamma = 2^2$  is 1.7dB compared to the MMSEC. The complexity is  $2^{12}$  in conventional ML detection.

Fig. 6 depicts the excess  $E_b / N_0$  to achieve  $\text{BER}=10^{-3}$  compared to  $K=1$  as a function of  $K$ . The performance is degraded as  $K$  increases due to increased self-interference.

The MMSEC and ML detection respectively have about 8 and 3dB degradation when multi-code transmission is fully employed ( $K=8$ ). It can be seen that the S-ML with  $\Gamma = 2^4$  provides near ML performance except  $K = 8$  (1dB degradation compared to ML with  $\Gamma = 2^{16}$ ).

Fig. 7 depicts the performance of detection schemes when BPSK modulation is employed without coding,  $L=8$  and  $K=6$ . As mentioned before, the use of the proposed spreading code can provide an additional performance improvement of about 1.2dB in the ML detection. Even S-ML ( $\Gamma = 2^2$ ) with the use of the proposed spreading code shows 1dB performance improvement than the conventional ML detection with conventional code. This performance improvement will be very meaningful for cell boundary users with BPSK modulation, since the improvement of performance directly increases the average rate of low-rate users.

## 5. Conclusion

In the cell-boundary environment, it is desirable to employ a transmission scheme that can provide performance robust to severe interference and low carrier to interference ratio. We consider the use of OFDM-CDM system with ML detection for cell-boundary environment. We have proposed complex-reduced ML detection schemes that can provide the performance with an acceptable loss compared to the optimum ML detector, while significantly reducing the complexity. Finally, a new spreading code is proposed for BPSK modulation so that the proposed scheme can be directly employed. Simulation results show that the use of new code improves the ML detection performance compare to the use of conventional code in BPSK modulation.

And it is shown that it outperforms ML detection with conventional code even though its complexity is considerably less than conventional ML detection. Therefore, the proposed schemes can enhance the system capacity of cell-boundary environment significantly with affordable receiver complexity.

## 6. Reference

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